Dissecting the NUT FACTOR

Engineers use a "nut factor" to relate a bolt's installation torque to the tension on it. But they need to understand the friction-related variables that contribute to the nut factor to accurately design bolted joints.

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Key points:

- The experimentally determined nut factor, *K*, consolidates the friction effects on threaded-fastener systems.
- A nut factor determined for one fastener geometry is valid for fasteners of similar geometry but different diameters.
- Testing joint prototypes highlights fastener interactions and points out design flaws.

Resources:

Archetype Joint LLC, *www.archetypejoint.com*

"Joint Decisions," MACHINE DESIGN, April 1, 2005, *http://tiny.cc/ZgGTy*

"How Tight is Right?" MACHINE DESIGN, August 23, 2001, *http://tiny.cc/tH8zc*

Measuring torque when installing threaded fasteners is the best indicator of future joint performance, right? Actually, bolt tension is a better performance indicator, but measuring torque is far easier to do.

Bolt tension is created when a bolt elongates during tightening, producing the clamp load that prevents movement between joint members. Such movement is arguably the most common cause of structural joint failures. The relationship between applied torque and the tension created is described by the relationship:

$$T = K \times D \times F$$

where T = torque, K = nut factor, sometimes called the friction factor, D = bolt diameter, and F = bolt tension generated during tightening. This expression is often called the short-form equation.

The nut factor

The nut factor, K, consolidates all factors that affect clamp load, many of which are difficult to quantify without mechanical testing. The nut factor is, in reality, a fudge factor, not derived from engineering principles, but arrived at experimentally to make the short-form equation valid.

Various torque-tension tests call for controlled tensioning of a threaded fastener while monitoring both torque and tension. At a specified torque or tension, the known values for T, D, and F are inserted into the short-form equation to permit solving for K.

Many engineers use a single value of K across a variety of threaded-fastener diameters and geometries. This approach is valid to some extent, because an experimentally determined nut factor is by definition independent of fastener diameter. But to truly understand the factors involved, it is helpful to compare the short-form equation with a torque-tension relationship derived from engineering principles.

Several of these equations are common, especially in designs that are primarily used in the E.U. Each produces similar results and takes the general form:

$$T = F \times X$$

where X represents a series of terms detailing fastener geometry and friction coefficients. These relationships are often referred to as long-form equations. (Three of the most widely used long-form equations are discussed in the sidebar, *The long way*.)

To understand how the nut factor compares to the terms in the long-form equations, let's consider the so-called Motosh equation:

$T_{in} = F_p \times [(P/2\pi) + (\mu_t \times r_t/\cos\beta) + (\mu_n \times r_n)]$

where T_{in} = input torque, F_p = fastener preload, P = thread pitch, μ_t = friction coefficient in the threads, r_t = effective radius of thread contact, β = half angle of the thread form (30° for UN and ISO threads), μ_n = friction coefficient under the nut or head, and r_n = effective radius of head contact.

The equation is essentially three terms, each of which represents a reaction torque. The three reaction torques must sum to equal the input torque. These elements, both dimensional and frictional, contribute in varying degrees to determining the torque-tension relationship, the purpose of calculating nut factor.

The impact of variables

So how do design decisions influence the nut factor that defines the torque-tension relationship? Specifically, engineers may wonder how valid a nut factor determined from a torque-tension test on one type of fastener is for other fastener geometries.

The short-form equation is structured so that the fastener diameter, D, is separate from the nut factor, K. This implies that a nut factor derived from torque-tension tests on one fastener diameter can be used to calculate the torque-tension relationship for fasteners with a different diameter.

The long way

Three of the most widely used long-form equations are shown below. The first equation was originally published in J. Bickford's *Introduction to the Design and Behavior of Bolted Joints, 2nd Ed.*, 1990, attributed to N. Motosh, 1976, and is usually referred to as the Motosh equation. The second equation is from DIN 946/VDI 2230. The third equation was published in ISO 16047 and is attributed to R. Kellerman and H. Klein, 1955.

$T_{in} = F_p \times [(P/2\pi) + (\mu_t \times r_t/\cos\beta) + (\mu_n \times r_n)]$

$M_A = F_V \times [(0.159 \times P) + (0.578 \times d_2 \times \mu_G) + (D_{Km} \times \mu_K / 2)]$

$T = F \times [0.5 \times (P + 1.154 \times \pi \times \mu_{th} \times d_2)/(\pi - 1.154 \times \mu_{th} \times P / d_2) + (\mu_b \times (D_o + d_h)/4)]$

The following table defines each variable and shows that, although the long-form equations appear unrelated, they are actually different forms of the same equation and yield similar results. Each term calculates a length which, when multiplied by the force generated by bolt tension, generates a moment or torque. They represent reaction torques resisting the input torque and must sum to equal that input torque.

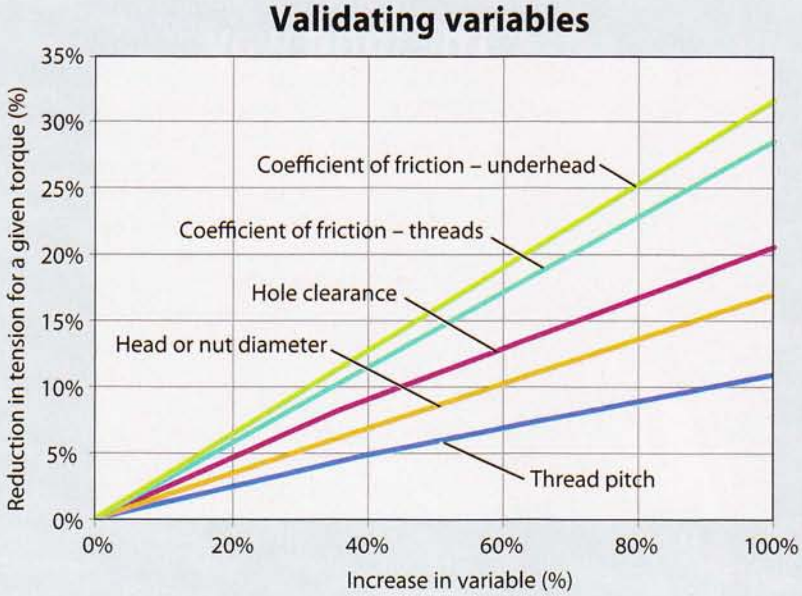
The term containing the variable P is the clamp load on thread pitch's inclined plane. The second term — involving μ_t, μ_G, or μ_{th} — is resisting torque caused by thread friction, and the last — using μ_n, μ_K, or μ_b — is a similar resisting torque generated by friction between the nut or head face and mating surface. The value of each term indicates the relative influence of each of these friction factors.

For example, applying the Motosh equation to an M12-1.75 flange-head screw with friction coefficients μ_t and μ_n equal to 0.15, shows that each additional 1,000 N of tension produces 0.28 N-m of reaction torque from clamp load on the thread pitch, 0.93 N-m of reaction torque from thread friction, and 1.35 N-m of reaction torque from friction under the nut. Total torque is then 2.56 N-m. These values break out to 10.9%, 36.3%, and 52.8%, respectively, of the total torque, confirming the commonly heard assertion that only 10 to 15% of the input torque goes toward stretching the bolt.

Motosh	DIN/VDI	ISO 16047	Description
T_{in}	M_A	T	Input torque
F_p	F_V	F	Fastener preload or tension
P	P	P	Thread pitch
r_n	$D_{Km}/2$	$(D_o + d_h)/4$	Effective radius of head contact
μ_n	μ_K	μ_b	Coefficient of friction under head
μ_t	μ_G	μ_{th}	Coefficient of friction in threads
r_t	$d_2/2$	$d_2/2$	Effective radius of thread contact (half the thread-pitch diameter)
β			Half-angle of thread form (30° for UN and ISO threads)

Like the nut factor itself, however, this approach is only an approximation. The accuracy of using a nominal fastener diameter, *D*, to apply a constant nut factor across a range of fastener sizes depends on the extent to which fastener diameter affects reaction torque.

Because some variables that affect bolted-joint design have more of an impact than others, it makes sense to examine them individually.



While the nut factor, *K*, in the short-form equation is experimentally determined, long-form equations use several variables to quantify the effects of thread pitch and friction on the torque-tension relationship. The traces shown here help explain the effect each variable has on bolt tension.

Clearance-hole diameter, for instance, is directly tied to nominal diameter. The long-form equations calculate that swapping a close-diametrical-clearance hole with one 10% larger leads to a 2% reduction in bolt tension for a given torque. Enlarging the fastener-head bearing diameter by 35%, say by replacing a standard hex-head fastener with a hex-flange-head, cuts bolt tension by 8% for a given torque.

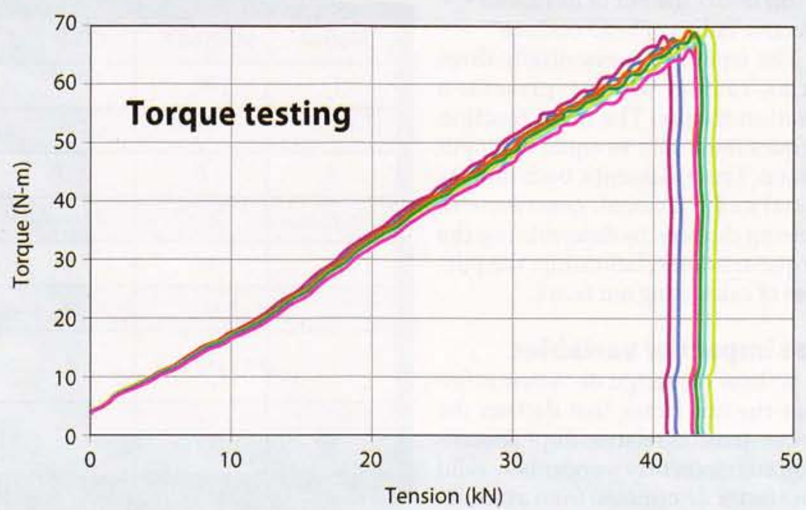
Both bearing diameter and hole clearance generally scale linearly with bolt diameter, so their relative importance remains the same over a range of fastener diameters. However, different head styles or clearances change the reaction torque from underhead friction because the contact radius changes.

Doubling the thread friction coefficient on its own, say by changing the finish or removing lubricant, reduces bolt tension by 28% for a given torque. Engineers should note that the thread and underhead friction coefficients are often assumed to be equal for convenience in test setups and calculations. ISO 16047 estimates that this assumption can lead to errors of 1 to 2%.

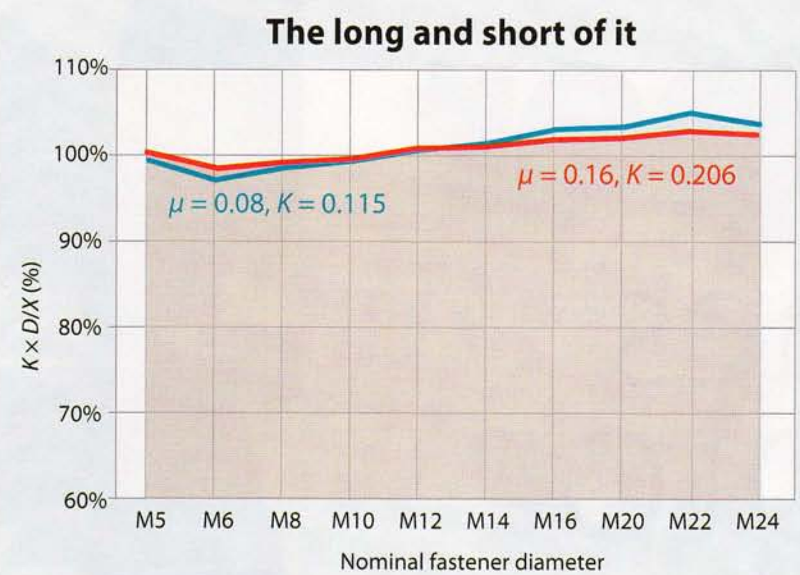
Thread pitch tends to be more independent of nominal diameter than the other variables. Increasing just thread pitch by 40% cuts tension 5% for a given torque. However, the reaction-torque term containing the thread pitch, $P - P/2\pi$ — does not contain a friction coefficient. Therefore, as fastener diameter and, consequently, friction increase, the relative importance of thread pitch falls.

There is a maximum deviation of 4.2% between the results of the short and long-form equations for standard-pitch, metric, hex-head-cap screws with constant and equal coefficients of friction. As friction coefficients increase, there's less error in assuming the nut factor varies directly with nominal diameter because the impact of thread pitch, the most independent variable, falls.

So, if all else is held constant, it's reasonable to apply a nut factor calculated at one fastener diameter across a range of fastener sizes. For best results, engineers should base testing on the weighted mean diameter of the fasten-



Torque-tension tests on simplified representative joints let engineers determine a nut factor that can define the torque-tension relationship for similar fasteners with different diameters. However, sample-to-sample variations in friction can make results vary by as much as 10%.



The short-form equation, $T = K \times D \times F$, using an experimentally derived nut factor, *K*, gives slightly different results than the long-form equation over a range of fastener sizes as shown here for two different friction levels. Results fall within 5% of each other. The values for μ and *K* were chosen to normalize traces for M12 fasteners.

ers for which the nut factor will be used.

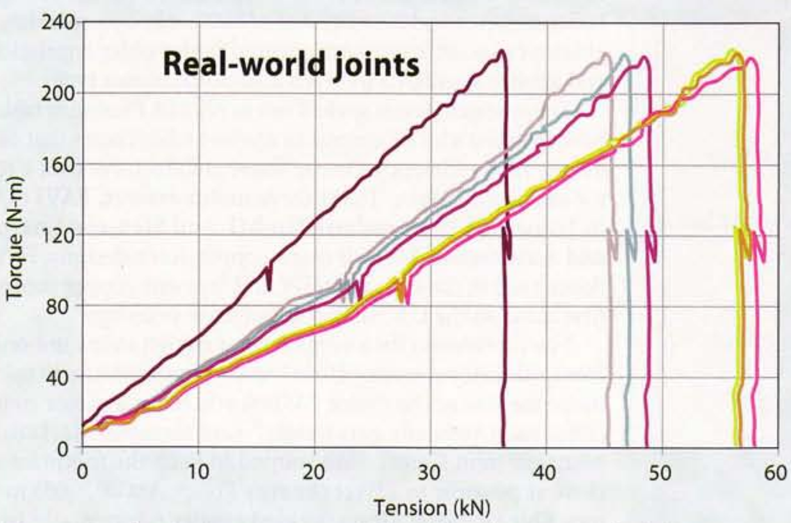
Many engineers find it expedient to apply a single nut factor across even greater fastener ranges, such as those with different head styles or clearance diameters. Applying the nut factor to joints where geometric variables other than nominal diameters are changing causes the short-form-equation results to differ from the long-form results by up to 15%.

Long or short?

So given all the discussion about which variables impact the nut factor and when it is reasonable to use a constant nut factor across a range of fasteners, it may seem as if the long-form equations get engineers closer to the "right" answer thanks to their fundamental correctness. However, long-form equations, even when derived from engineering principles, use assumptions and approximations that may make them no more correct than an equation using an experimentally derived nut factor.

The potential errors in both types of equations pale compared to the variations in real-world joints. Benchtop torque-tension testing shows approximately 10% variations within samples even when all fastener and bearing materials are the same. According to both short and long-form equations, there should be no variation at all. Friction coefficients seem to vary from sample to sample.

The situation is even worse in production-representative joints. One in-joint torque-tension test with bolt tension measured in real time using ultrasonic pulse-echo techniques revealed variations inherent in how components fit together. A six-bolt pattern magnified the effects of imperfect contact between bolts and the bearing surface and produced a 60% sample-to-sample variation because both geometric and friction variables were changing over the sample set.



Torque-tension tests on prototype joints, like this one on a six-bolt pattern, can highlight design problems in joint design. In this case, variations of up to 60% from sample to sample indicated that the joint geometry was magnifying the effects of imperfect contact between fastener and bearing surface.

The bottom line is that neither the nut factor nor the long-form equation's friction coefficients can be reliably established using reference tables. Only testing accurately determines friction conditions. As we have seen, these are too sensitive to component and assembly variation to be determined by analysis alone.

Testing reliably converts input torque to induced tension, letting engineers determine mean friction coefficients or nut factors and the distributions about those means. Tests may also help engineers identify inherent shortcomings in joints that need to be revised to make them more reliable. **MD**