Comparison of Analytical Methods and Test Results for Determining Fastener Elongation Part 1

by Dave Archer

Objective

Analytical analysis of bolted joints requires either the direct calculation of fastener elongation or calculation of the components that determine elongation. **Even in instances when tension can be measured directly during testing, calculating elongation can be desirable when direct measurement is impractical.** For example, when the clamp load at conditions not tested are desired, calculation of elongation or its components are required to extrapolate the test data. A primary challenge of calculating elongation is translating the stress profile in the head and the engaged threads into equations that can be solved simply. As opposed to the portion of the shank that lies within the grip length, these transition areas can contain complex stress profiles. This is further complicated by the fact that it is the head and nut member that contain most of the features that differentiate the various types of threaded fasteners. As a result there are several published approaches to elongation calculation that contain subtle differences for solving this problem. In this paper we will use two popular calculation methods to compare analytical estimates to actual test measurements of elongation. In light of the challenge of estimating the contribution of the head and nut areas, a test matrix was created to test various head/nut combinations, including some that might not represent practical application. Because the transition areas become more influential as the grip length get shorter tests were conducted at two different ratios of grip length to diameter. In this first phase of testing fully threaded cap screws were used so that the shank would be a constant cross section and the effect of the various head and nut transitions tested could be preserved. Each test was run at two loads as a check of elongation measurement by reviewing the ratio of high/low values.

Calculation Methods

The two estimation methods used as a comparison to tested values of elongation are those from Bickford[1] and VDI 2230. In their simplest form of relating force to elongation, both methods are identical:

伸长率分析法与测试结果比较 Part 1

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目的

螺栓零件的分析牵涉到主要「直接运算紧固件的伸长率」，以及针对「各种影响伸长率因素的运算」二个部分，虽然张力数值可使用仪器直接量测，但当无法进行直接测量时，伸长率运算则是必须的，如还没经过实际测试，就希望取得夹具负载值时，伸长率的运算并公布推测试资料是必要，伸长运算的主要困难点在於「如何将螺头与咬合螺纹的压力特性，转换为容易计算的公式」，相较小於握固长度（grip length）的螺身，这些接合区的压力特性可能相当复杂。此外，不同种类的螺拴和螺帽，具有各种不同特徵，又会让问题更加复杂。要计算紧固件伸长率有许多方法，而这些不同方法间又有些许的差异，本文采用两种常见的计算方式，来比较预测值与实际测试值。此外，为了预估螺头与螺帽的组合，本文另外设计了一种测试矩阵，用来计算各种头型（不论常见与罕用）与螺帽组合，因接触区域会受到握固长度缩短影响，因此测试时会采用两种不同的握固长度与直径比例，测试的第一部份采用「完全螺纹」螺丝，因其螺身可保持固定的截面，以避免不同的头型与螺帽接触时的差异，且每次测试都会进行两次，以检查高/低比例之伸长率差异。

y = FL / EA where:

y = elongation F = force/tension L = length

E = modulus of elasticity A = area

But this relationship is only valid for samples with a constant cross-section, so the relationship is expanded to handle the fact that portions of a fastener have different cross sections.

$y = F\delta b$ where: δb = bolt resilience = $\delta h + \delta t + \delta n = Lh/EAh + Lt/EAt + Ln/EAn$

Bolt resilience (the inverse of stiffness) consists of three terms: head resilience, thread resilience and nut resilience. It is in the length and area variables of these terms where the differences in the two estimates lie.

First, the VDI method expands the nut resilience in to two terms which replaces δ_n. These terms separate the contribution of the nut and the engaged threads.

$\delta ne = Ln/EAn + Le/EAe$

The remaining differences are summarized below.

Test Method

Figure 1 summarizes the test matrix chosen for the test. English fasteners were chosen because all configurations of fully threaded metric bolts and jamb nuts were not readily available in a common property class. Figure 2 provided better visualization of the test variants. Load was applied with a universal test machine (UTM). Elongation was measured with a pair of lever style gage heads contacting the ends of the fastener. The gage heads were supported from a common point on the UTM's fixed platen. The gages output to an amplifier set to read at 0.00005" resolution.

Test Results

Test results are tabulated in Figures 3A and 3B and graphed in Figures 4A and 4B. The test set-up appeared to be reasonably robust as the ratio between elongation at high and low loads for the six tests in which they were both run was 1.99, 2.06, 2.03, 2.03, 1.96 and 2.07. The average deviation from test values for the Bickford and VDI 2230 calculations was 7.6% and 9.4%,

运算方法

这两种比较伸长率的计算方法，源於Bickford[1]与VDI 2230，这两种方法的施力/伸长率简单关连式是相同的。

y＝FL / EA where： y＝伸长率；F＝施力/张力；L＝长度；E＝弹性模量；A＝面积

不过这个关系式只有在「固定截面积」时才有效，因此，为因应紧固件不同部分的不同截面积，计算的关系式进一步扩展为：

$y = F\delta_b$ where：

δ_b＝螺栓预紧力 (bolt resilience)＝$\delta_h + \delta_t + \delta_n = L_h/EA_h + L_t/EA_t + L_n/EA_n$

螺栓预紧力（刚度的相反）包含螺头预紧力、螺纹预紧力、以及螺帽预紧力三个部分，长度、面积等变数，造成两种预估值之间的差异。

首先，VDI法延伸螺帽预紧力到两个变数，取代了 δ_n，这些变数区隔了螺帽与咬合螺纹。

$\delta_{ne} = L_n/EA_n + L_e/EA_e$

其他的差异摘要如下表：

Variable 变数	Bickford	VDI 2230
L_h	$0.5h_h$	0.4d
A_h	a_s [1]	a_d
L_t [2]	l_g	l_g
A_t [2]	a_s	a_s
L_n , L_e	$0.5h_n$	0.4d , 0.5d
A_n , A_e	a_s	a_d , a_s

h_h = head height 头高
h_n = nut height 螺帽高
a_s = stress area (at minor dia)
压力面积（较小直径）
a_d = area of nominal dia 名义上直径面积
l_g = grip length 握固长度
d = nominal diameter 名义上直径

[1]Use a_d when shank is not fully threaded

[2]This test uses fully threaded bolts. In cases where there is an unthreaded portion of the shank δ_t is broken into a threaded and unthreaded term.

[1]如螺身没有完全螺纹化则采用 ad

[2]该测试采用完全螺纹螺栓，因为部分螺身有螺纹而部分没有，因此螺身 δt 分解成螺纹区与非螺纹区

Fig. 1 - **Test Matrix**

Test #	Fastener	L/D	Nut 1	Nut 2
1	Hex Cap Screw - Long	5.78	Tapped Block	N/A
2	Hex Cap Screw - Long	6.80	Hex	N/A
3	Hex Cap Screw - Long	6.31	Jamb	N/A
4	Stud	4.59	Hex	Tapped Block
5	Stud	5.81	Hex	Hex
6	Hex Cap Screw - Short	1.20	Tapped Block	N/A
7	Hex Cap Screw - Short	1.70	Hex	N/A
8	Hex Cap Screw - Short	2.55	Jamb	N/A

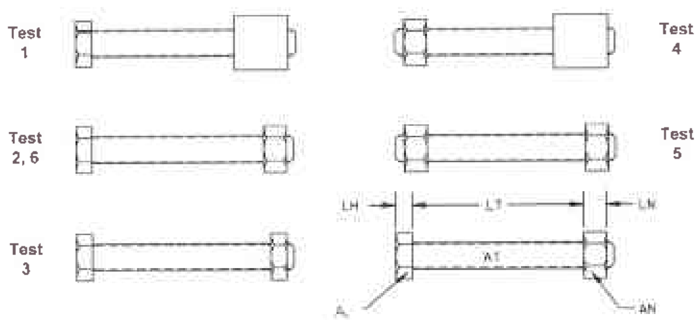
Notes: 1. All fasteners SAE J429 Gr 8 except tapped block which is AISI 1018
2. All fasteners 3/8-16 UNC fully threaded
3. Tapped block 0.75" deep
4. All tests run with sample size of 6
5. A settling run was performed before the measurment run
6. Load was applied at a rate of 16,000 lb/min
7. L/D represents the ratio of grip length to nominal fastener dia
8. Applied loads: 7,440 lb and 3,720 lb
9. Modulus of Elasticity: 29.3x10^6 lb/in^2

表1- 测试矩阵

测试编号	紧固件	L/D	螺帽1	螺帽2
1	六角螺丝- 长	5.78	自攻防松	无
2	六角螺丝- 长	6.80	六角	无
3	六角螺丝- 长	6.31	边框	无
4	螺柱	4.59	六角	自攻防松
5	螺柱	5.81	六角	无
6	六角螺丝- 长	1.20	自攻防松	无
7	六角螺丝- 长	1.70	六角	无
8	六角螺丝- 长	2.55	边框	无

备注: 1. 所有符合SAE J429 8级紧固件（除AISI 1018的自攻防松紧固件）
2. 所有符合UNC 3/8-16紧固件都完全螺纹化
3. 自攻防松0.75吋深
4. 6种尺寸样本都接受所有测试
5. 在测量开始之前要先进行处理过程
6. 负载力采16,000 lb/min
7. L/D代表握固长度除以名义上紧固件直径
8. 采用负载: 7,440 lb 和 3,720 lb
9. 弹性模量: 29.3x10^6 lb/in^2

Fig. 2 - **Test Configurations** 图2 – 测试组态



测试方法

表1为测试矩阵的摘要，在此测试采用英制紧固件，因在常用类型中，缺少采用公制完全螺纹螺栓与螺帽。图2是视觉化测试结果，施力采用通用测试机（UTM）来进行，一对杠杆式量测头，置於紧固件两端进行伸长率的测量，量测头由UTM的固定盘固定点所支撑，量测值输出到设定为0.00005"解析度之放大器。

测试结果

测试结果在表3A与3B 列表，并在图4A与4B图示。测试环境显然相当稳固，六次高低负载、测试的伸长

Figure 3A - **Elongation at 40% Proof Load**
图 **3A - 40%** 施力之伸长率

Test Number 测试编号	Elongation 伸长率, in x 10-3			Relative to Test 差异比例	
	Bickford	VDI 2230	Test	Bickford	VDI 2230
1	4.35	4.20	4.73	-8.0%	-11.2%
2	4.63	4.83	4.76	-2.8%	1.5%
3	4.24	4.53	4.47	-5.1%	1.4%
4	3.68	3.47	4.12	-10.7%	-15.7%
5	4.08	4.22	4.57	-10.8%	-7.7%
6	1.54	1.39	1.63	-5.1%	-14.4%
7	1.50	1.70	1.51	-0.7%	12.6%
8	1.93	2.22	2.38	-19.2%	-6.9%

Figure 3B - **Elongation at 80% Proof Load**
图 **3B - 80%** 施力之伸长率

Test Number 测试编号	Elongation 伸长率, in x 10-3			Relative to Test 差异比例	
	Bickford	VDI 2230	Test	Bickford	VDI 2230
1	8.71	8.41	9.43	-7.6%	-10.8%
2	9.25	9.66	9.81	-5.7%	-1.6%
3					
4	7.36	6.95	8.36	-11.9%	-16.9%
5	8.15	8.44	9.28	-12.2%	-9.0%
6	3.08	2.78	3.18	-3.1%	-12.6%
7	2.99	3.40	3.13	-4.2%	8.7%
8					

High loads not applied to jamb nut (tests 3 and 8)
jamb螺帽并未进行高负载测试（测试3与8）

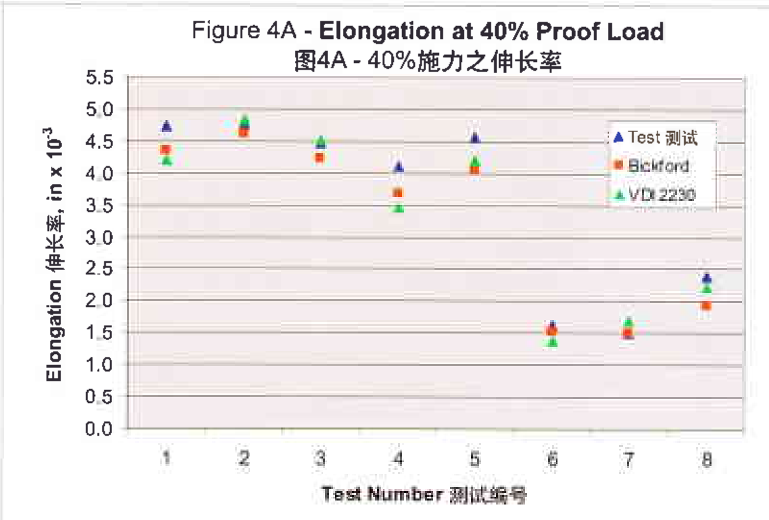


Figure 4A - **Elongation at 40% Proof Load**
图4A - 40%施力之伸长率

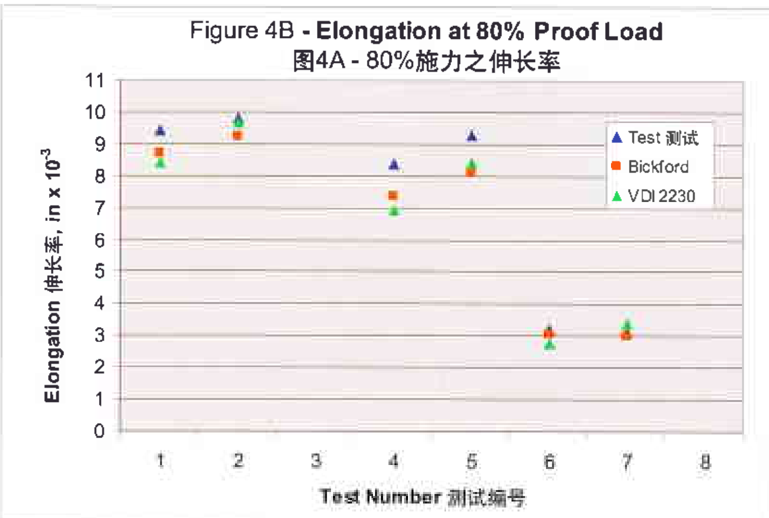


Figure 4B - **Elongation at 80% Proof Load**
图4A - 80%施力之伸长率

respectively. Estimating elongation independent of thread engagement seems to have affected correlation of VDI estimates with tests utilizing the tapped block. It is not immediately apparent why the calculations didn't correlate very well with tests conducted with a stud.

Next Steps

Our plan for the next phase of this study is to gain better fidelity of the relative contributions of the shank and the nut and head by taking extensometer readings on the shank, including those that are partially unthreaded.

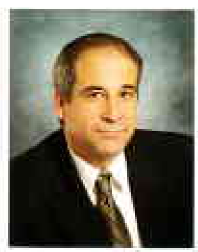
[1]Ref: An Introduction to the Design and Behavior of Bolted Joints, 2nd Edition by John Bickford.

率比例分别为1.99、2.06、2.03、2.03、1.96与2.07。Bickford与VDI 2230运算的平均变异分别为7.6% 与 9.4%。因伸长率与螺纹咬合两者独立，似乎在测试螺丝块时影响了VDI的相关性。至於为何与螺栓（stud）相关的测试结果相关性不高，原因并不很清楚。

下一步

本研究下一阶段的计画，是在螺身与部分螺纹的部分采用引伸计（extensometer），以提高螺身、螺帽、头型测量的正确性。

[1]参考文献：An Introduction to the Design and Behavior of Bolted Joints, 2nd Edition by John Bickford.



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Dave Archer is President and founder of Archetype Joint LLC, an Orion, Michigan company devoted exclusively to joint design, testing, and validation. To support this capability Archetype Joint maintains an A2LA accredited test lab. Customers include automotive, heavy vehicle, defense, aerospace, industrial equipment, and recreational product manufacturers.

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